2012 American Control Conference
Fairmont Queen Elizabeth, Montréal, Canada
June 27-June 29, 2012

Influencing Emotional Behavior in a Social Network

[1]Z. Kan, [2]J. M. Shea, and [1,2]W. E. Dixon
[1]Department of Mechanical and Aerospace Engineering,
[2]Department of Electrical and Computer Engineering,
University of Florida, Gainesville, FL 32611-6250, USA.
Email: kanzhen0322@ufl.edu, jshea@ece.ufl.edu, wdixon@ufl.edu

Abstract— **Mankind has always sought out social interaction, and our social interactions influence our thoughts and actions. As technological advances in social media provide a means for more rapid, convenient, and widespread communication, our resulting social interactions can lead to a more dynamic influence. Underlying these interactions are emotional responses to different stimuli and a desire not to become isolated from peers. In this paper, such social interactions are modeled as an undirected graph where each vertex represents an individual and each edge represents a social bond between individuals. Motivated by the non-local property of fractional-order systems, the emotional response of individuals in the network is modeled by fractional-order dynamics whose states depend on influences from social bonds. A decentralized control method is then developed to manipulate the social group to a common emotional state while maintaining existing social bonds (i.e., without isolating peers in the group). Mittag-Leffler stability methods are used to prove asymptotic convergence to a common equilibrium point (i.e., emotional state) of the networked fractional-order system.**

I. INTRODUCTION[1]

Mankind has always sought out social interaction, and our social interactions influence our thoughts and actions. As technological advances in social media provide a means for more rapid, convenient, and widespread communication, our resulting social interactions can lead to a more dynamic influence. Flash mobs are being organized through social media for events ranging from entertaining public spontaneity to vandalism, violence, and crime [1]–[3]. In addition to flash mobs, recent riots and protests [4]–[7] and ultimately revolution [8], [9], have been facilitated through social media technologies such as Facebook, Twitter, You Tube, and BlackBerry Messaging (BBM).

In attempts to prevent, mitigate, or prosecute the sources of such social unrest, governments and law enforcement agencies are placing a greater emphasis on examining (and ultimately controlling) the structure of social networks. Scotland Yard is looking to social media posts as part of investigations into widespread looting and rioting in London [6], [7], and police in San Francisco disabled access to social networks by cutting off cellphone service as a means to prevent riots due to a police shooting [5]. One U.S. Intelligence strategy in Afghanistan is to focus on answering rudimentary questions about Afghanistan's social and cultural fabric through tools such as Nexus 7 to tap into the exabytes of information "for leveraging popular support and marginalizing the insurgency" [10]. Yet other's argue that Nexus 7 lacks models or algorithms.

Models and algorithms have been extensively developed for various engineered networks and multi-agent systems [11], [12]. Consensus is a particular class of network control problem that has been extensively studied where the goal is for the individual nodes to reach an agreement on the states of all agents [13], [14]. However, an interesting question that has received little attention is how can such models and methods be applied toward understanding and controlling a social network. How can one produce consensus among a social network (e.g., to manipulate social groups to a desired end)? Motivated towards this end, the focus in this paper is to influence the emotions of a socially connected group of individuals to a consensus state.

Various dynamic models have been developed for psychological phenomena, including efforts to model the emotional response of different individuals [15]–[17]. A dynamic model of love is reported in the work of [15], which describes the time-variation of the emotions displayed by individuals involved in a romantic relationship. In [16], happiness is modeled by a set of differential equations, and the time evolution of one's happiness in response to external inputs is examined. A mathematical model of fear is also described in the work of [17].

Fractional-order differential equations are a generalization of integer-order differential equations which exhibit a non-local property where the next state of a system not only depends upon its current state but also upon its historical states starting from the initial time [18]. This property has motivated researchers to explore the use of fractional-order systems as a model for various phenomena in natural and engineered systems, and in relation to the current context, have also been explored as a potentially more appropriate model of psychological behavior. For example, the integer-order dynamic models of love and happiness developed in [15] and [16] were revisited in [19] and [20], where the models were generalized to fractional-order dynamics, since a

[1]This research is supported by AFRL Munitions Directorate and the AFRL Collaborative System Control STT.

person's emotional response is influenced by past experiences and memories. However, the results in [15], [16], [19], [20] focus on an individual's emotion model, without considering the interaction among individuals in the context of a social network where rapid and widespread influences from the social peers can prevail.

Instead of studying an individual model of a person's emotional response, the work here aims to investigate and influence the interaction of a person's emotions within a social network. Using graph theory, a social network is modeled as an undirected graph, where an individual in the social network is represented as a vertex in the graph, and the social bond between two individuals is represented as an edge connecting two vertices. Motivated by the non-local property of fractional-order systems, the emotional response of individuals in the network are modeled by fractional-order dynamics whose states depend on influences from social bonds. Within this formulation, the social group is modeled as a networked fractional system. The first apparent result that investigated the coordination of networked fractional systems is [21], in which linear time invariant systems are considered and where the interaction between agents is modeled as a link with a constant weight. In this paper, the social bond between two persons is considered as a weight for the associated edge in the graph measuring the closeness of the relationship between the individuals. In comparison to [21], the main challenge in this work is that social bonds are time varying parameters which depends on the emotional states of individuals. Previous stability analysis tools such as examining the Eigenvalues of linear systems for fractional-order systems (cf. [20]–[22]) are not applicable to the time-varying system in this work. This paper also considers a social bond threshold on the ability of two people to influence each other's emotions. To ensure interaction among individuals, one objective is to maintain existing social bonds among individuals above the prespecified threshold all the time (i.e., social controls or influences should not be so aggressive that they isolate or break bonds between people in the social group). Another objective is to design a distributed controller for each individual, using local emotional states from social neighbors, to achieve emotion synchronization in the social network (i.e., an agreement on the emotion states of all individuals). To achieve these objectives, a decentralized potential function is developed to encode the control objective of emotion synchronization, where maintenance of social bonds is modeled as a constraint embedded in the potential function. Asymptotic convergence of each emotion state to the common equilibrium in the social network is then analyzed via a Metzler Matrix [23] and a Mittag-Leffler stability [24] approach.

II. Preliminaries

A. Fractional Calculus

Consider the fractional order nonautonomous system

$$ {}_{t_0}^{C}D_t^{\alpha} x(t) = f(t,x) \tag{1} $$

with initial condition $x(t_0)$, where ${}_{t_0}^{C}D_t^{\alpha} f(t)$ denotes the Caputo fractional integral of order $\alpha \in (0,1)$ on $[t_0, t]$, and $f(t,x)$ is piecewise continuous in t and locally Lipschitz in x. Caputo and Riemann-Liouville (R-L) fractional derivatives are the two most widely used fractional operators [18]. Since the R-L fractional operator requires a fractional-order initial condition, which can be difficult to interpret [25], the subsequent development is based on the Caputo fractional operator. Stability of the solutions to (1) are defined by the M-L function as follows [24].

Definition 1: (Mittag-Leffler Stability) The solution of (1) is said to be Mittag-Leffler stable if

$$ \|x(t)\| \leq \{m[x(t_0)] E_{\alpha,1}(-\lambda (t-t_0)^{\alpha})\}^{b}, $$

where t_0 is the initial time, $\alpha \in (0,1)$, $b > 0$, $\lambda > 0$, $m(0) = 0$, $m(x) \geq 0$, $m(x)$ is locally Lipschitz, and $E_{\alpha,1}$ is defined as $\sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha+1)}$ with $z \in \mathbb{C}$ and the Gamma function $\Gamma(\cdot)$.

Lyapunov's direct method is extended to fractional-order systems in the following Lemma to determine Mittag-Leffler stability for the solutions of (1) in [24].

Lemma 1: [24] Let $x = 0$ be an equilibrium point for the system (1), and $\mathbb{D} \subset \mathbb{R}^n$ be a domain containing the origin. Let $V(t,x) : (0,\infty] \times \mathbb{D} \to \mathbb{R}$ be a continuously differentiable function and locally Lipschitz with respect to x such that

$$ \begin{aligned} k_1 \|x\|^{a} &\leq V(t,x) \leq k_2 \|x\|^{ab}, \\ {}_{0}^{C}D_t^{\beta} V(t,x) &\leq -k_3 \|x\|^{ab}, \end{aligned} $$

where $x \in \mathbb{D}$, $\beta \in (0,1)$, k_1, k_2, k_3, a and b are arbitrary positive constants. Then $x = 0$ is Mittag-Leffler stable.

B. Graph Theory

Graph theory (see cf. [26]) is widely used to represent a networked system. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote an undirected graph, where $\mathcal{V} = \{v_1, \cdots, v_N\}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denote the set of nodes and the set of edges, respectively. Each edge $(v_i, v_j) \in \mathcal{E}$ represents the neighborhood of node i and node j, which indicates that node i and node j are able to access each other's states. The neighbor set of node i is denoted as $\mathcal{N}_i = \{v_j \mid (v_i, v_j) \in \mathcal{E}\}$. A path between v_1 and v_k is a sequence of distinct nodes starting with v_1 and ending with v_k such that consecutive nodes are adjacent in graph $\mathcal{G}$. Graph $\mathcal{G}$ is connected if in $\mathcal{G}$ any node can be reached from any other node by following a series of edges. The adjacency matrix is defined as $A \triangleq [a_{ij}] \in \mathbb{R}^{N\times N}$ with $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. A matrix L for the graph $\mathcal{G}$ is defined as $L \triangleq A - D \in \mathbb{R}^{N\times N}$, where $D \triangleq [d_{ij}] \in \mathbb{R}^{N\times N}$ is a diagonal matrix with $d_{ii} = \sum_{j=1}^{N} a_{ij}$. The $N \times N$ matrix with positive or zero off-diagonal elements and zero row sums is referred as a Metzler matrix [27]. Zero is a trivial eigenvalue of a Metzler matrix, and all the other eigenvalues are positive, if and only if the corresponding undirected graph $\mathcal{G}$ is connected. The eigenvector associated with the zero eigenvalue is $\mathbf{1}$, where $\mathbf{1} = [1, \cdots, 1]^T \in \mathbb{R}^N$.

To facilitate the following development, a corollary to Theorem 1 of [23] is introduced as follows.

Corollary 1: The equilibrium point $x^* \mathbf{1} \in \mathbb{R}^N$ of the system

$$\dot{x}(t) = L(t)\, x(t) \tag{2}$$

is exponentially stable (i.e., the elements of $x(t) \in \mathbb{R}^N$ achieve exponential consensus), provided that the time-varying matrix $L(t) \in \mathbb{R}^{N\times N}$ in (2) is a Metzler matrix with piecewise continuous and bounded elements, and the time-varying graph corresponding to $L(t)$ is connected for all $t \geq 0$.

III. Problem Formulation

Consider a social network composed of N individuals. Using graph theory, the interaction among individuals is modeled as an undirected fixed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. For instance, the karate club network in [28] is modeled as an undirected graph as shown in Fig. 1, where the vertex in the graph $\mathcal{G}$ is represented by an individual, the solid arrow connecting two individuals denotes the edge in $\mathcal{G}$, representing an established social bond (i.e., friendship) and indicating that the individuals are able to access each other's states (i.e., sense and understand the social state of a peer).

In a social network, the state of an individual can be the social status, social connections, emotional status, or etc. In the following development, the social state denotes some human emotion, such as happiness, love, anger or fear. The emotion state $q_i(t) \in \mathbb{R}$ is a real number indicating the current state of an individual i that can be detected from other members (i.e., social neighbors such as close friends or family) in the social network. For instance, a greater value of $q_i(t)$ implies a happier state of individual i.

An integer derivative of a function is only related to its nearby points, while a fractional-order derivative involves all the previous points. Since human emotions are always influenced by memories and past experiences, $q_i(t)$ is modeled as the solution to a fractional-order dynamic as

$${}_0^C D_t^\alpha q_i(t) = u_i(t),\ i = 1, \cdots, N, \tag{3}$$

where $u_i \in \mathbb{R}$ denotes an influence (i.e., control input) over the emotional state, and ${}_0^C D_t^\alpha q_i(t)$ is the α^{th} derivative of $q_i(t)$ with $\alpha \in (0, 1]$. The model in (3) is a heuristic approximation to an emotional response. The model indicates that a person's emotional state is a direct relationship to external influence integrated over the history of a person's previous emotional states. On-going efforts by the scientific community are focused on the development of clinically derived models; yet, to date, there is no widely accepted model of a person's emotional response to events in a social network.

Social bonds in a network can be established through a number of relationships between individuals (e.g., student and teacher, employer and employee, patient and doctor, two strangers that share a common interest) and can be represented as an undirected edge in graph $\mathcal{G}$. Each bond has a weighting factor denoted as $S_{ij} \geq 0$ that measures the amount of influence that is shared between individuals i and j. The greater the value S_{ij}, the closer the relationship between individuals i and j, and $S_{ij} = 0$ if two individuals have no influence over each other. Through an analysis of a social graph over time, one could determine a weighting for the level of influence between individuals. However, the subsequent development only requires that an individual node has an understanding of the relative influence between itself and its local social neighbors. Moreover, it is assumed that, there exist a threshold $\delta \in \mathbb{R}^+$, and individuals i and j are able to influence each other's emotional states only when the social bond $S_{ij} \geq \delta$. In other words, an edge ε_{ij} in graph $\mathcal{G}$ does not exist if the social bond S_{ij} between individuals i and j is less than the threshold δ. The neighbors of individual i in graph $\mathcal{G}$ is defined as $\mathcal{N}_i = \{v_j \mid S_{ij} \geq \delta\}$, which determines a set of individuals who have an influential relationship with individual i. In the subsequent development, the social bond is defined as

$$S_{ij} = f\left(\|q_i - q_j\|^2\right), \tag{4}$$

where $f(\cdot)$ is a differentiable function, mapping the emotion states of individuals i and j to a real non-negative value. Some properties for S_{ij} include: 1) $f\left(\|q_i - q_j\|^2\right)$ decreases as $\|q_i - q_j\|$ increases (the further apart the emotional state of two individuals the less influence they have over each other), which indicates that $\frac{\partial f}{\partial \|q_i - q_j\|} < 0$; 2) $f\left(\|q_i - q_j\|^2\right)$ achieves the minimum of 0 when individual i has no influence/relationship with individual j; 3) the second partial derivative $\frac{\partial^2 f}{\partial q_i}$ is bounded. These properties are based on the general observation that the emotional states of individual i and j tend to consensus in a close relationship. For example, the emotional state of one spouse, parent or child tends to mirror the emotional state of another spouse, child, or parent respectively. Hence, it is reasonable to assume that S_{ij} is a function of the difference between q_i and q_j, designed as $\|q_i - q_j\|^2$ in this work. While some discrete events can cause a discontinuous shift in someone's social bonds (e.g., a cheating spouse, winning the lottery, unexpected sickness or death) that would lead to an unbounded second partial derivative, most social bonds tend to be continuous over time.

Based on the problem setting, the social network of human emotions is now formulated as a networked fractional-order system described by (3). The emotion synchronization objective in a social network is to regulate the emotional states of individuals to a desired state (i.e., $q_i(t) \to q^*$ for all i with $q^* \in \mathbb{R}$ denoting an equilibrium point). Moreover, individuals generally prefer to share an emotional response rather than react in an emotional way that renders them an outcast. Hence, the emotion synchronization problem also includes a goal that given an initially connected graph $\mathcal{G}$, the social bonds between individuals are maintained (i.e., maintain the social bonds $S_{ij} \geq \delta$ all the time so that peers remain peers). Since social bonds exist initially, any two individuals are able to reach each other through a path of edges associated with a social bond satisfying $S_{ij} \geq \delta$.

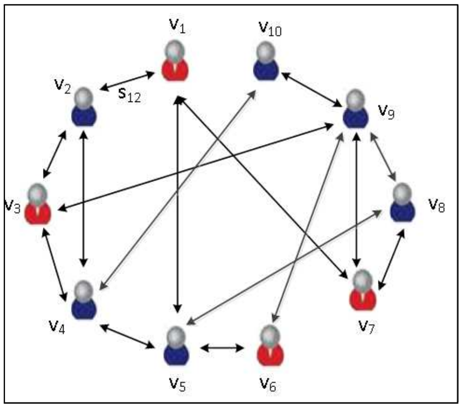

Fig. 1. The Zachary's karate club network in [28] is modeled by an undirected graph $\mathcal{G}$, where the numbered vertex in the graph represents the members of the club, and solid line connecting two individuals denotes the established social bond (i.e., friendship) in the club.

IV. Control Design

Artificial potential field based methods, composed of attractive and repulsive potentials, have been widely used in the control of multi-agent systems, where the control objective is encoded as the minimum potential value by the attractive potential and constraints are encoded as the maximum potential value by the repulsive potential (cf. [29], [30]). Driven by the negative gradient of the proposed potential field, the system will asymptotically achieve the minimum of the potential field. In this work, the potential field approach is applied to social control.

To achieve emotion synchronization, a decentralized potential function is developed as $\varphi_i : \mathbb{R}^N \to [0,1]$ for individual i (of N) as

$$\varphi_i = \frac{\gamma_i}{\left(\gamma_i^k + \beta_i\right)^{1/k}}, \tag{5}$$

where $k \in \mathbb{R}^+$ is a tuning parameter, $\gamma_i : \mathbb{R}^2 \to \mathbb{R}^+$ is the goal function, and $\beta_i : \mathbb{R}^N \to \mathbb{R}^+$ is a constraint function. The goal function in (5) is designed as

$$\gamma_i = \sum\nolimits_{j\in\mathcal{N}_i} \frac{1}{2}\left\|q_i - q_j\right\|^2, \tag{6}$$

which is minimized whenever the emotional state of individual i agrees with the emotions of neighbors j, $j \in \mathcal{N}_i$. To ensure existing social bonds are maintained (i.e., $S_{ij} \geq \delta$), the constraint function in (5) is designed as

$$\beta_i = \prod\nolimits_{j\in\mathcal{N}_i} \frac{1}{2} b_{ij}, \tag{7}$$

where $b_{ij} = S_{ij} - \delta$, and S_{ij} is defined in (4). For an existing social bond between individuals i and j, the potential function φ_i in (5) will approach its maximum whenever the constraint function β_i decreases to 0 (i.e., the social bond S_{ij} decreases to the threshold of δ).

Based on the definition of the potential function in (5), the emotional influence is designed as

$$u_i = -K_i \nabla_{q_i}\varphi_i, \tag{8}$$

where K_i is a positive gain. In (8), $\nabla_{q_i}\varphi_i$ denotes the gradient of φ_i with respect to q_i, as

$$\nabla_{q_i}\varphi_i = \frac{k\beta_i \nabla_{q_i}\gamma_i - \gamma_i \nabla_{q_i}\beta_i}{k(\gamma_i^k + \beta_i)^{\frac{1}{k}+1}}. \tag{9}$$

From (6) and (7), $\nabla_{q_i}\gamma_i$ and $\nabla_{q_i}\beta_i$ in (9) can be determined as

$$\nabla_{q_i}\gamma_i = \sum\nolimits_{j\in\mathcal{N}_i} \left(q_i - q_j\right), \tag{10}$$

and

$$\begin{aligned}\nabla_{q_i}\beta_i &= \sum\nolimits_{j\in\mathcal{N}_i} \bar{b}_{ij}\frac{1}{2}\nabla_{q_i} b_{ij} \\ &= \sum\nolimits_{j\in\mathcal{N}_i}\left(\frac{\partial f}{\partial\left(\left\|q_i - q_j\right\|^2\right)}\right)\bar{b}_{ij}\left(q_i - q_j\right),\end{aligned} \tag{11}$$

respectively, where $\bar{b}_{ij} \triangleq \prod_{l\in\mathcal{N}_i, l\neq j} b_{il}$. Substituting (10) and (11) into (7), $\nabla_{q_i}\varphi_i$ is rewritten as

$$\nabla_{q_i}\varphi_i = -\sum\nolimits_{j\in\mathcal{N}_i} m_{ij}\left(q_i - q_j\right), \tag{12}$$

where

$$m_{ij} = \frac{k\beta_i - \bar{b}_{ij}\gamma_i\left(\frac{\partial f}{\partial\left(\left\|q_i - q_j\right\|^2\right)}\right)}{k(\gamma_i^k + \beta_i)^{\frac{1}{k}+1}} \tag{13}$$

is non-negative, based on the first property of S_{ij}, and the definition of γ_i, β_i, k, and $\bar{b}_{ij}$.

V. Convergence Analysis and Social Bond Maintenance

To show that individuals in the fractional-order network converge to a common desired emotional state, the following analysis is segregated into three proofs. In the first proof, the connectivity of the social group is proven to be ensured by the influence function in (8). In the second proof, an integer-order simplification of the dynamic system in (3) is considered and exponential convergence is proven. Exponential convergence of the integer-order system is used to establish the existence of a Lyapunov function and its derivative by invoking a converse Lyapunov theorem. The Caputo fractional derivative of the developed Lyapunov function is then determined and used within a Mittag-Leffler stability analysis that proves the closed-loop fractional-order system asymptotically converges to the equilibrium set of consensus states.

A. Social Bond Maintenance

Assuming a social network is initially connected, the social group will remain connected if every existing edge in the network graph is maintained all the time (i.e., $S_{ij} \geq \delta$). The following Lemma is developed to show that connectivity of the underlying graph is maintained under the influence function in (8) (i.e., social peers do not become isolated and disconnected from the social group).

Lemma 2: The influence function in (8) guarantees connectivity of $\mathcal{G}$ all the time.

Proof: Consider an emotional state q_0 for individual i, where the bond between individual i and neighbor $j \in \mathcal{N}_i$

satisfies $b_{ij}(q_0, q_j) = 0$, which indicates that the social bond is too weak to affect the emotion of individual i, and the associated edge is about to break. From (7), $\beta_i = 0$ when $b_{ij} = 0$, and the navigation function φ_i achieves its maximum value from (5). Since φ_i is maximized at q_0, no open set of initial conditions can be attracted to q_0 under the negated gradient control law designed in (8). Therefore, the social bond between individual i and j is maintained greater than δ by (8), and the associated edge is also maintained. Following similar arguments, every edge in $\mathcal{G}$ is maintained, and connectivity of the underlying graph is guaranteed. ∎

B. Convergence Analysis

For the particular case of $\alpha = 1$, the fractional-order dynamics in (3) simplifies to the integer-order system $\dot{q}_i(t) = u_i(t)$. The following theorem establishes exponential convergence to the common equilibrium for the integer-order system.

Theorem 1: The equilibrium point $q^* \in \mathbb{R}$ of the initially connected graph of nodes with integer-order dynamics $\dot{q}_i(t) = u_i(t)$ is exponentially stable for all i, given the influence function $u_i(t)$ developed in (8).

Proof: For $\alpha = 1$, substituting (8) and (12) into (3) yields the following closed-loop emotion dynamics of individual i:

$$\dot{q}_i(t) = -\sum\nolimits_{j\in\mathcal{N}_i} K_i m_{ij}(q_i - q_j). \tag{14}$$

Using (14) and similar to [31], the dynamics of all individuals in the social network can be rewritten in a compact form as

$$\dot{\mathbf{q}}(t) = \pi(t)\,\mathbf{q}(t), \tag{15}$$

where $\mathbf{q} = \begin{bmatrix} q_1, & \cdots, & q_N \end{bmatrix}^T$ denotes the stacked vector of q_i, and the elements of $\pi(t) \in \mathbb{R}^{N\times N}$ are defined as

$$\pi_{ik}(t) = \begin{cases} -\sum_{j\in\mathcal{N}_i} K_i m_{ij} & i = k \\ K_i m_{ij} & j \in \mathcal{N}_i, i \neq k \\ 0, & j \notin \mathcal{N}_i, i \neq k. \end{cases} \tag{16}$$

From (16), $\pi(t)$ is matrix with zero row sums. Using the fact that m_{ij} is non-negative from (13), and K_i is a positive constant gain in (8), the off-diagonal elements of $\pi(t)$ are positive or zero, and its row sums are zero. Hence, $\pi(t)$ is a Metzler matrix. Given that $\pi(t)$ is a Metzler matrix and the social network is always connected with the controller developed in (8) (see Lemma 2), Corollary 1 can be applied to (15) to conclude that the elements of $\mathbf{q}(t)$ exponentially achieve consensus. ∎

Theorem 2: The equilibrium point $q^* \in \mathbb{R}$ of the initially connected graph of nodes with the fractional-order dynamics in (3) with $\alpha \in (0,1)$ is asymptotically stable for all i, given the influence function $u_i(t)$ developed in (8).

Proof: Let $x_i(t) \triangleq q_i(t) - q^* \in \mathbb{R}$ and $\mathbf{x}(t) \triangleq \mathbf{q}(t) - q^*\mathbf{1} \in \mathbb{R}^n$. The fractional-order dynamics in (3) with $\alpha \in (0,1)$ for all individuals can be obtained from (15) as

$${}_0^C D_t^{\alpha}\mathbf{x}(t) = \pi(t)(\mathbf{x}(t) + q^*\mathbf{1}) \triangleq g(t,\mathbf{x}), \tag{17}$$

where $\pi(t)$ is the same as in Theorem 1, since each element in $\pi(t)$ is a function of $q_i(t) - q_j(t)$ from (13) and $q_i(t) - q_j(t) = x_i(t) - x_j(t)$. Since the stability of a fractional-order system is defined by Definition 1, and Mittag-Leffler stability implies asymptotic stability as discussed in [24], the following development is focused on proving that (17) is Mittag-Leffler stable.

Since γ_i and β_i are not zero simultaneously, and γ_i, β_i and their partial derivatives are bounded from (10) and (11), $\pi(t)$ in (17) is bounded. Assuming that $\pi(t)$ is bounded by a constant $l \in \mathbb{R}^+$, the Lipschitz condition for $g(t,\mathbf{x})$ in (17) is

$$\frac{\|g(t,\mathbf{x})\|}{\|\mathbf{x}\|} \leq l. \tag{18}$$

Theorem 1 states that the equilibrium point q^* is exponentially stable for the integer-order system of (15). The converse Lyapunov theorem, Theorem 4.9 in [32], indicates that there exists a function[2] $V(t,\mathbf{x}) : (0,\infty] \times \mathbb{R}^N \to \mathbb{R}$ and strictly positive constants k_1, k_2, and k_3 such that

$$k_1\|\mathbf{x}\| \leq V(t,\mathbf{x}) \leq k_2\|\mathbf{x}\|, \tag{19}$$

$$\dot{V} \leq -k_3\|\mathbf{x}\|. \tag{20}$$

Let $\beta = 1 - \alpha \in (0,1)$. Following a similar procedure in the proof of Theorem 8 in [24] and using (18) and (20), the Caputo fractional derivative of V is computed as

$$\begin{aligned} {}_0^C D_t^{\beta} V(t,\mathbf{x}) &= {}_0^C D_t^{1-\alpha} V(t,\mathbf{x}) = {}_0^C D_t^{-\alpha}\dot{V} \\ &\leq -k_3\left({}_0^C D_t^{-\alpha}\|\mathbf{x}\|\right) \\ &\leq -k_3\left({}_0^C D_t^{-\alpha}\frac{\|g(t,\mathbf{x})\|}{l}\right) \\ &\leq -\frac{k_3}{l}\left\|{}_0^C D_t^{-\alpha} g(t,\mathbf{x})\right\| \\ &\leq -\frac{k_3}{l}\|\mathbf{x}\|. \end{aligned} \tag{21}$$

Mittag-Leffler stability of system (17) with $\alpha \in (0,1)$ can be obtained as

$$x(t) \leq \frac{V(0, x(0))}{k_1} E_{1-\alpha}\left(-\frac{k_3}{k_2 l} t^{1-\alpha}\right), \tag{22}$$

by applying Lemma 1 to (19) and (21), where $a = b = 1$. The result in (22) implies the equilibrium point $q^*\mathbf{1} \in \mathbb{R}^n$ of the closed-loop fractional-order system in (17) is asymptotically stable. ∎

VI. Discussion

The previous development is based on the assumption that q^* is a common equilibrium point for all the individuals in a social network. In some situations, a common equilibrium point for an emotional state (e.g., group anger) could be derived from a discrete event (e.g., a police shooting [6], [7]) or long term events (e.g., years of oppression from a dictator [8], [9]). In such situations, the controller in (8) provides instructions for an individual to combine emotional differences with social peers, while considering the strength of the respective social bonds, so that as the individual's

[2] As discussed in [23], one valid selection for the Lyapunov function is $V(x) = \max\left\{ x_1, \cdots, x_n \right\} - \min\left\{ x_1, \cdots, x_n \right\}$.

emotional state converges to q^*, social bonds (i.e., the need for peers to share an emotional state) between peers will also influence them to converge to the same emotional state. If a person instantly converges to q^*, the emotional difference between social peers may decrease to the point where $S_{ij} < \delta$, resulting in a separation from the social group and an end of the individual's influence over the group (i.e., the change in emotional state is great enough that bonds between social peers are broken and the social peers ignore the individual's state). The controller in (8) accounts for the weighted interactions and influence over peers based on the assumption that peers will integrate an emotional state in a non-local fractional-order sense.

Of course, individuals in a social network often do not have a common equilibrium point. For example, a group of friends may wish to engage in an activity that differs from the desire of an individual. In these scenarios, a person must resolve the conflict between the individual equilibrium point and the social bond constraint that $S_{ij} \geq \delta$. That is, either peer pressure will deviate the person from the desired social state, strengthening corresponding social bonds, or social bonds with the group will decrease/break. This observation indicates that long term peers with strong social bonds likely share a common equilibrium point. Follow-on efforts to the current work are being developed to incorporate the dynamics of the equilibrium point/social bond arbitration along with influence strategies to enable social peers to deviate a person from an equilibrium, or change the equilibrium.

VII. Conclusion

In this work, emotion synchronization for a group of individuals in a social network is studied. By modeling human emotion as a fractional-order system, a decentralized potential field-based function is developed to ensure that the emotion states of all individuals asymptotically converge to a common equilibrium while maintaining social bonds. Social bonds play an import role in a person's emotional state. For instance, a person tends to put greater trust in a close friend than some random person, and thus, can be more easily influenced by the close friend. However, the current development only examines the social bond as a threshold constraint to ensure continued interaction between friends, without considering the potential dynamics of how a person's emotions can be affected by different social bonds in the network. Hence, future work is being considered that explores the relation between a person's emotion and the associated different levels of social bonds. Moreover, further efforts are also targeting influence strategies to enable social peers to deviate a person from an equilibrium, or change the equilibrium (i.e., peer pressure strategies applied to reluctant peers).

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